A Stochastic Modelling Approach for Simultaneous Demands in Domestic Water Supply Systems

L.T. Wong and K.W. Mui

Department of Building Services Engineering
The Hong Kong Polytechnic University
Hong Kong, China
Plumbing systems in high-rise buildings: Demand characteristics (1)

A washroom:
- Washbasin
- Shower
- Water closet (?)
- Other appliance(s)

balance between cost and service level in the design of an efficient water supply system for high-rise residential buildings
Plumbing systems in high-rise buildings: Demand characteristics (2)

- **Low demand probability** at each of the installed appliances
- **Relatively steady flow rate** at the appliance
- Design is not economically justified for the possible maximum simultaneous demands
- Current Practice: design can only address the simultaneous demand most of the time
- Fixture unit approach
Current practice (1)

- Characterize an appliance: flow rate $q_a$, discharge probability $p = t/T$ (average time fraction)
- Assume service level, 4 persons or 8 persons per washroom
- Assume per-person utilization rate in the peak hours
- Assume constant flow rate
- Assume independent use of each appliance
- Assume binominal operation, (on-off mode)
Evaluate for the density function of the number $N$ of reference appliances in operations simultaneously, for $M$ total installed reference appliances

$$M p_N = \frac{M!}{N!(M - N)!} p^N (1 - p)^{M-N}$$

* Solve for $N$ at $\lambda$

$$\lambda = p(N+1) + p(N+2) + \cdots + p(M-1) + p(M) = \sum_{i=N+1}^{M} p_i ; N < M$$
Current practice (3)

- Using Sterling’s formula and expanding in descending powers of $M$ gives:

\[
\log p = \log M! + (Mp+x)\log p + [(1-p)M-x] \log(1-p) - \log(Mp+x)! - \log[(1-p)M-x]!
\]

\[
= \frac{1}{2} \log 2\pi (1-p) - \frac{1}{2M} \left\{ \frac{x^2}{p(1-p)} + \frac{x(1-2p)}{p(1-p)} \right\} + \ldots
\]

- … solution: \[ N = Mp + k \lambda \sqrt{2Mp(1-p)} \]
- Currently acceptable failure rate is 1%, $k \lambda = 1.8$
Assign an equivalent fixture unit for any other appliance characterized by \( q \) and \( p \), at an reference hypothetical flow rate of 10 L/s

\[
\text{Fixture unit} = \frac{M_{\text{ref}}}{M_{(p,q)}}
\]
Current practice (5): re-consideration for domestic washroom

- Characterize an appliance: flow rate $q_a$, discharge probability $p = t/T$ (average time fraction)
- Assume constant flow rate
- Assume independent use of each appliance
- The demand is occupant load dependent, usage rate dependent
A proposed stochastic approach

- Taking the simultaneous usage patterns in some high-rise residential buildings into account, this study investigated the probable maximum demands of some water supply systems for domestic washrooms in high-rise residential buildings in Hong Kong using a time-flow rate approach.

- Recent field measurements → usage patterns
  → water supply flow rates

- Monte Carlo simulations → demand pattern
  → Probable maximum simultaneous demands
**Time-flow rate approach (1)**

\[
q_j(t) = \sum_i q_i(t) \quad t \in \tau_p = k_p \left( \tau_0 + \sum_i N_{o,i} \tau_i \right)
\]

- Probable flow rate of an operating appliance \(i\)
- Hourly probable number of operations
- Instantaneous water demand for washroom \(j\)
- Total time in the peak period with zero flow rate
- Discharge duration
- Only 1 appliance will be operating at one moment

For a washroom
Time-flow rate approach (2)

\[ q_d \approx \sum_{j=1}^{N_j} q_j \]

- \( q_d \) is the plant probable maximum simultaneous demand (L/s).
- \( N_j \) is the number of domestic washrooms.
- \( q_j \) is the water demand of each washroom \( j \) (L/s).
Time-flow rate approach (3)

The transient domestic washroom demand patterns within certain peak periods can be determined using Monte Carlo simulations, taking variations of $N_p$, $q_i$, $\tau_i$ and $n_i$ into account,

\[ N_{o,i} = N_p n_i \]

Step 1:

\[ N_p \in \tilde{N}_p; q_i \in \tilde{q}_i; \tau_i \in \tilde{\tau}_i; n_i \in \tilde{n}_i = \tilde{n}_i(t) \]
**Time-flow rate approach (4): cumulative discrete probability density function for flow rate at \( t \in \tau_p \)**

\[ \varphi \in (1,0): \]

\[ p_2 < \varphi < p_3 \]

\[ \alpha_0 = (0,0) \]

\[ \alpha_1 = (0, p_1) \]

\[ \alpha_2 = (q_1, p_1) \]

\[ \alpha_3 = (q_1, p_2) \]

\[ \alpha_4 = (q_2, p_2) \]

\[ \alpha_5 = (q_2, p_3) \]

\[ \alpha_6 = (q_3, p_3) \]

\[ \alpha_{2N-1} = (q_{N-1}, p_N) \]

\[ \alpha_{2N} = (q_N, p_N) \]

\[ \alpha_{2N+1} = (q_{N+1}, p_{N+1}) \]

**Step 2:**

\[ q_j = \begin{cases} 
0 & 0 \leq \varphi < p_1 \\
q_i & p_i \leq \varphi < p_{i+1} \\
q_N & p_N \leq \varphi \leq 1 
\end{cases} \]
An illustrative example (1)

- A survey of water demand patterns in 596 domestic washrooms referred
- average number of users per washroom = 4.2 (head count, or denoted as ‘hd’).

![Diagram showing diurnal demand patterns of some domestic appliances](image)

Figure 1: Diurnal demand patterns of some domestic appliances
### An illustrative example (2): Typical demand patterns of domestic washroom appliances

<table>
<thead>
<tr>
<th>Survey parameter</th>
<th>Normal distribution</th>
<th>Geometric distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AM (ASD)</td>
<td>Goodness-of-fit p-value</td>
</tr>
<tr>
<td>Cistern volume $V_{wc}$ (L)</td>
<td>9.2 (1.1)</td>
<td>&gt;0.01</td>
</tr>
<tr>
<td>Operating time $\tau_{wc}$ (s)</td>
<td>101 (55)</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>

#### Water closet (WC)

#### Washbasin

<table>
<thead>
<tr>
<th></th>
<th>Normal distribution</th>
<th>Geometric distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate $q_{wb}$ (L/s)</td>
<td>0.17 (0.07)</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Operating time $\tau_{wb}$ (s)</td>
<td>16.0 (18.6)</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>

#### Shower

<table>
<thead>
<tr>
<th></th>
<th>Normal distribution</th>
<th>Geometric distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate $q_{sh}$ (L/s)</td>
<td>0.31 (0.07)</td>
<td>&lt;0.02</td>
</tr>
<tr>
<td>Operating time $\tau_{sh}$ (min)</td>
<td>13.7 (7.5)</td>
<td>≥0.0001</td>
</tr>
</tbody>
</table>
An illustrative example (3): Input parameters

- Demand for all appliances between midnight and 5:00 a.m. was very low;
- Peak ‘per capita’ hourly demands occurred in the evening:
  - WC:
    - 6:00 p.m. to 9:00 p.m. - average 0.59/hd/h (ASD=0.20/hd/h)
      - maximum 1.25/ hd/h;
  - Washbasin:
    - 8:30 p.m. - average 0.59/ hd/h (ASD=0.21/ hd/h)
      - maximum 1.25/ hd/h;
  - Shower:
    - 7:00 p.m. - average 0.18/ hd/h (ASD=0.09/ hd/h)
      - maximum 0.29/ hd/h.
An illustrative example (4): Input parameters

- For simulations, e.g. a washroom:
- For appliances $i=3$ (i.e. a WC, a washbasin and a shower)
- assuming their respective peak demand profiles with an average occupant load per appliance of 4.2 hd ($ASD=1.0$ hd)
An illustrative example (5): Input parameters

Based on the time-flow rate approach, the design flow rates $q_d$ (L/s) due to a number of domestic washrooms were evaluated via Monte Carlo simulations for two cases:

1. none of the three appliances in a washroom would be used at the same time;
2. WC cistern and washbasin in a washroom would be used at the same time.

Employing the arbitrarily selected profiles from 2000 simulated washrooms, together with the expected appliance profiles:

- expected flow rate $q_i = \langle \tilde{q}_i \rangle$
- expected operating time $\tau_i = \langle \tilde{\tau}_i \rangle$
- expected occupant load $N_p = \langle \tilde{N}_p \rangle$
- during the recorded peak period $n_i = \tilde{n}_i(t)|_{\text{max}(N_o,d)}$
Simulation results (1)

**Graph Description:**
- The graph illustrates the relationship between the number of washrooms ($N_j$) and the probable maximum simultaneous demand ($q_d$) in liters per second ($L/s$).
- The x-axis represents the number of washrooms, while the y-axis shows the probable maximum simultaneous demand.
- The graph includes data points for different cases:
  - **Fixture unit approach** indicated by a triangle (△).
  - **Case (1), Geometrical distribution** indicated by a circle (○).
  - **Case (2), Geometrical distribution** indicated by a dot (●).
  - **Normal distribution** indicated by an 'X' (×).
  - **Expected profile** indicated by a plus sign (+).

The trend lines suggest an increasing demand as the number of washrooms increases.
Presently, hypothetical simultaneous usage patterns of domestic washroom appliances are used in the design of a water supply system which might not optimize the estimated demand for water supply system in some high-rise residential buildings of Hong Kong.
Concluding remarks (2)

- This study proposed a stochastic model of water demands in domestic washrooms for some high-rise residential buildings in Hong Kong using a time-flow rate approach while taking account of the simultaneous usage patterns of occupant loads, per-occupant demand rate, water flow rate and demand time of installed appliances.
Concluding remarks (3)

- With an illustrative example for Hong Kong case, this paper presented a template for the development of a stochastic demand model that estimates the probable maximum simultaneous water demands for high-rise residential buildings.
Acknowledgment

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