A Stochastic Modelling Approach for Simultaneous Demands in Domestic Water Supply Systems

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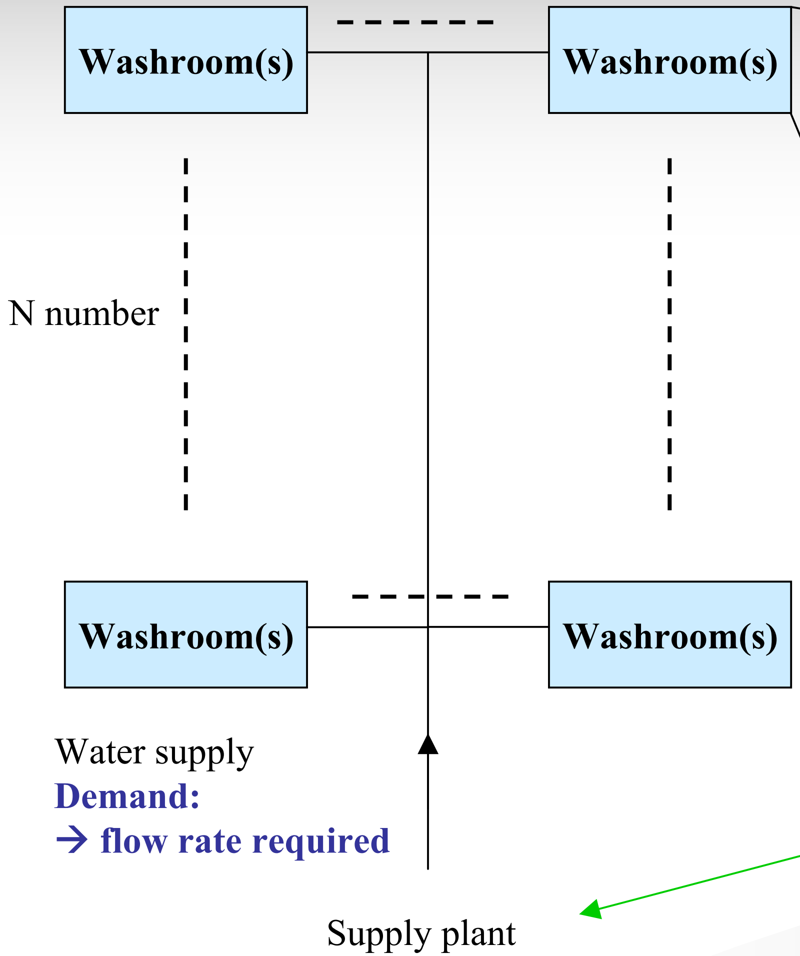
Plumbing systems in high-rise buildings: Demand characteristics (1)



Water supply
Demand:
→ flow rate required

A washroom:

- Washbasin
- Shower
- Water closet (?)
- Other appliance(s)

- balance between cost and service level in the design of an efficient water supply system for high-rise residential buildings

Plumbing systems in high-rise buildings: Demand characteristics (2)

- Low demand probability at each of the installed appliances
- Relatively steady flow rate at the appliance
- Design is not economically justified for the possible maximum simultaneous demands
- Current Practice: design can only address the simultaneous demand most of the time
- Fixture unit approach

Current practice (1)

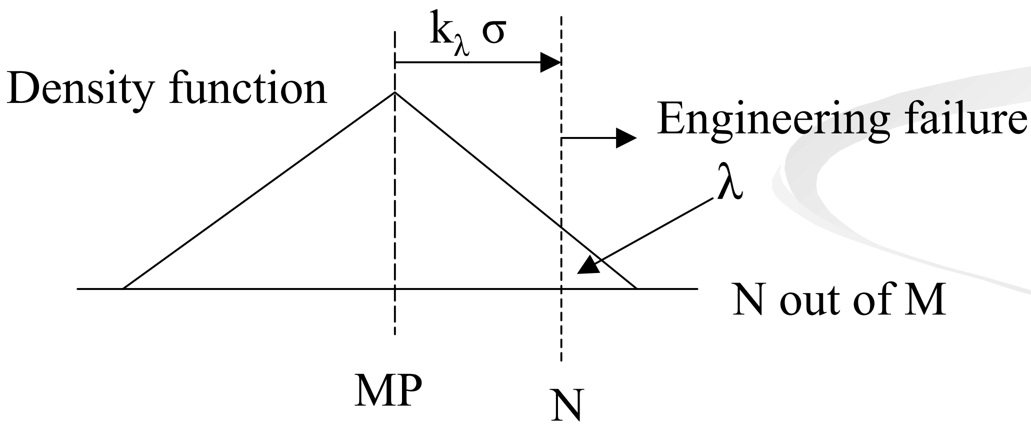
- Characterize an appliance: flow rate q_a, discharge probability $p = t/T$ (average time fraction)
- Assume service level, 4 persons or 8 persons per washroom
- Assume per-person utilization rate in the peak hours
- Assume constant flow rate
- Assume independent use of each appliance
- Assume binominal operation, (on-off mode)

Current practice (2)

- Evaluate for the density function of the number N of reference appliances in operations simultaneously, for M total installed reference appliances

$$ {}_{M}p_{N} = \frac{M!}{N!(M-N)!} p^{N} (1-p)^{M-N} $$

* Solve for N at λ



$$ \lambda = p(N+1) + p(N+2) + \cdots + p(M-1) + p(M) = \sum_{i=N+1}^{M} p_i \quad ; N<M $$

Current practice (3)

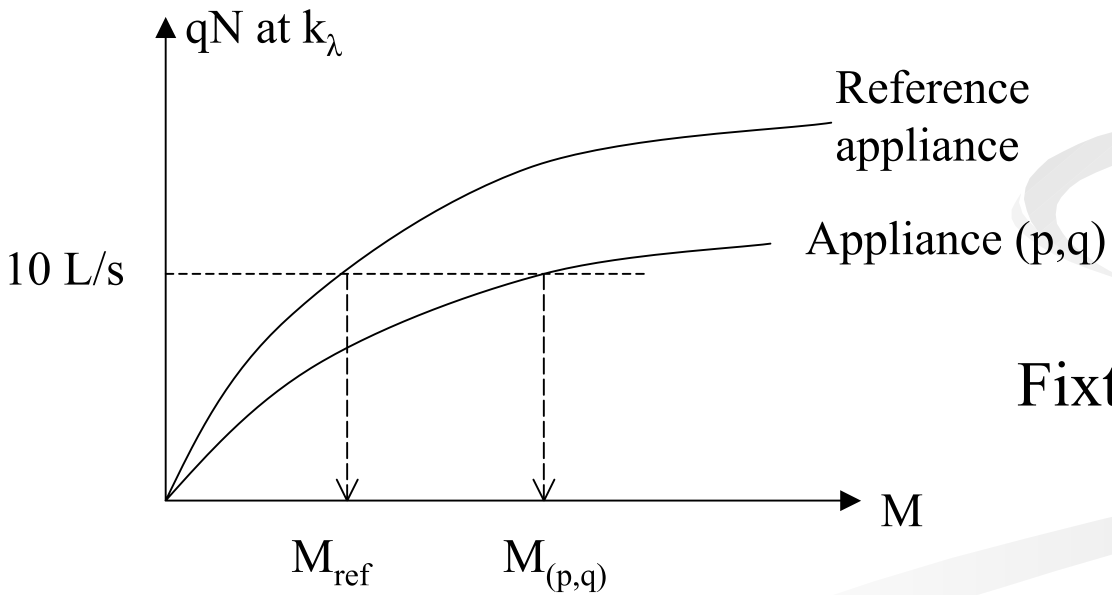
- Using Sterling's formula and expanding in descending powers of M gives:

$$\log p = \log M! + (Mp+x)\log p + [(1-p)M - x]\log(1-p) - \log(Mp+x)! - \log[(1-p)M-x]!$$
$$= \frac{1}{2}\log 2\pi p(1-p)M - \frac{1}{2M}\left\{\frac{x^2}{p(1-p)} + \frac{x(1-2p)}{p(1-p)}\right\}\cdots$$

- … solution: $N = Mp + k_\lambda \sqrt{2Mp(1-p)}$
- Currently acceptable failure rate is 1%, $k_\lambda = 1.8$

Current practice (4)

- Assign an equivalent fixture unit for any other appliance characterized by q and p, at an reference hypothetical flow rate of 10 L/s



Fixture unit = $M_{ref} / M_{p,q}$

Current practice (5): re-consideration for domestic washroom

- ☒ Characterize an appliance: flow rate q_a, discharge probability $p = t/T$ (average time fraction)
- ☒ Assume constant flow rate
- ☒ Assume independent use of each appliance
- ☒ The demand is occupant load dependent, usage rate dependent

A proposed stochastic approach

- Taking the simultaneous usage patterns in some high-rise residential buildings into account, this study investigated the **probable maximum demands** of some water supply systems for domestic washrooms in high-rise residential buildings in Hong Kong using a **time-flow rate approach**.
- Recent field measurements → usage patterns
  → water supply flow rates
- Monte Carlo simulations →demand pattern
  →Probable maximum simultaneous demands

Time-flow rate approach (1)

probable flow rate of an operating appliance i

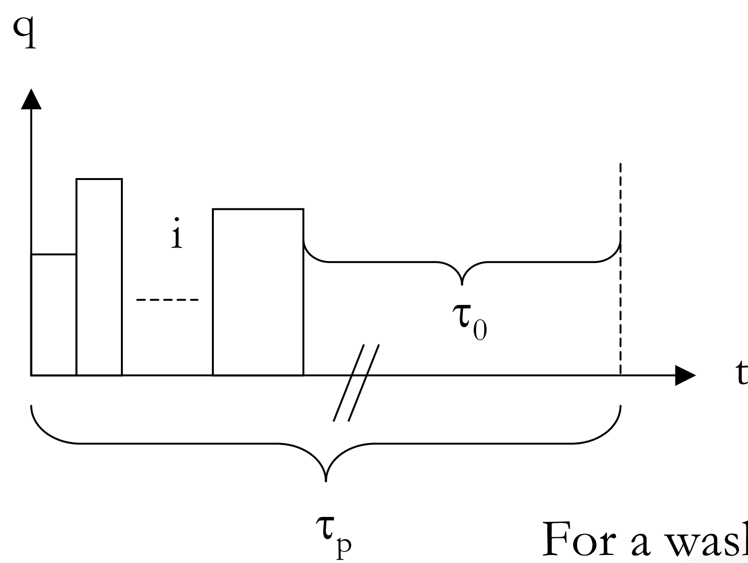
hourly probable number of operations

instantaneous water demand for washroom j

$$q_j(t)=\sum_i q_i(t) \qquad t\in\tau_p = k_p\left(\tau_0+\sum_i N_{o,i}\,\tau_i\right)$$

total time in the peak period with zero flow rate

discharge duration



For a washroom

Only 1 appliance will be operating at one moment

Time-flow rate approach (2)

number of domestic washrooms

the plant probable maximum simultaneous demand (L/s)

$$q_d \approx \sum_{j=1}^{N_j} q_j$$

water demand of each washroom j (L/s)

Time-flow rate approach (3)

number of users served
by the appliance i

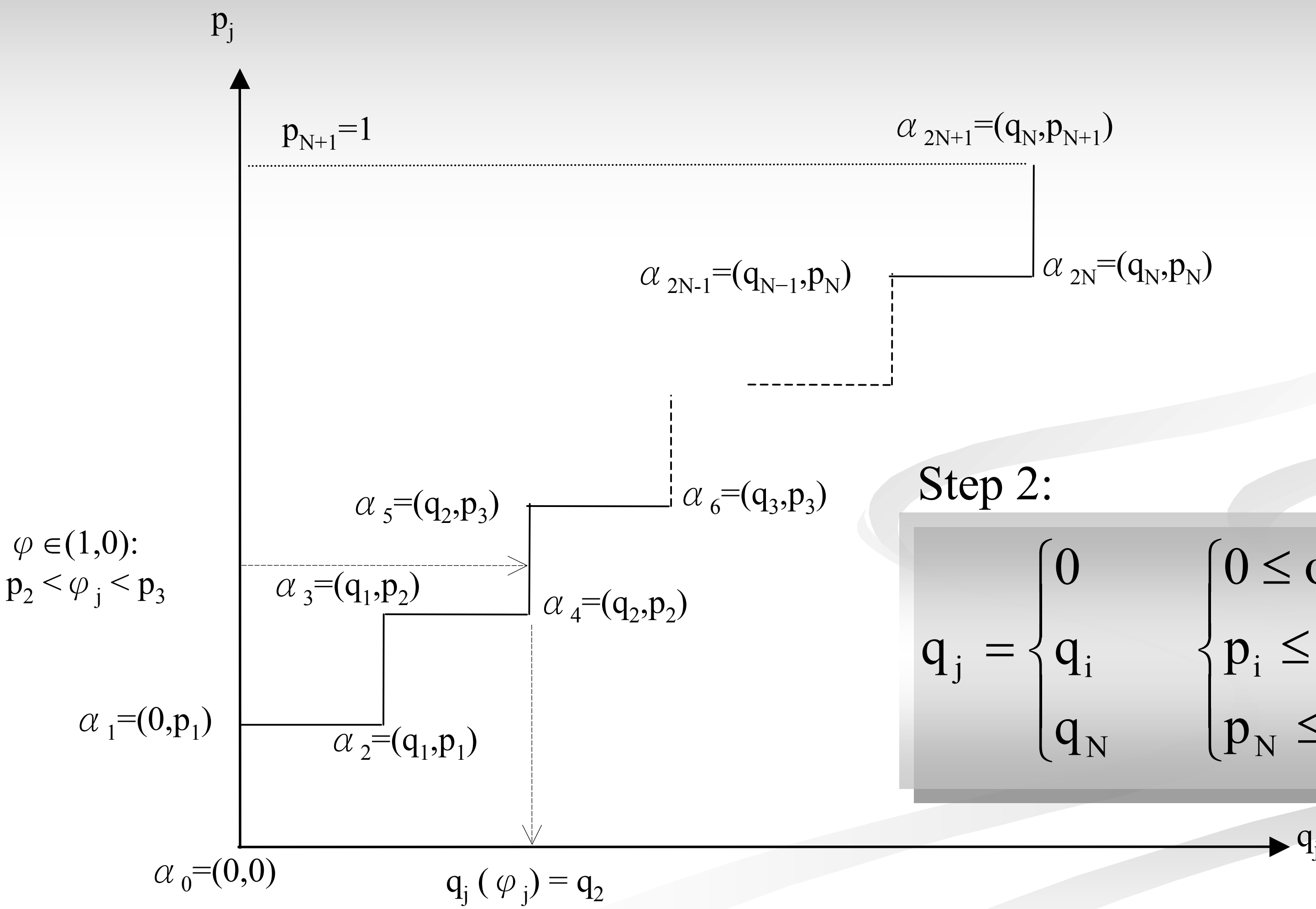
number of *per capita*
hourly operations

$$N_{o,i} = N_p n_i$$

- the transient domestic washroom demand patterns within certain peak periods can be determined using Monte Carlo simulations, taking variations of N_p, q_i, τ_i and n_i into account,

Step 1: $$N_p \in \tilde{N}_p ; q_i \in \tilde{q}_i ; \tau_i \in \tilde{\tau}_i ; n_i \in \tilde{n}_i = \tilde{n}_i(t)$$

Time-flow rate approach (4): cumulative discrete probability density function for flow rate at $t \in \tau_p$



Step 2:

$$q_j = \begin{cases} 0 & 0 \le \varphi < p_1 \\ q_i & p_i \le \varphi < p_{i+1} \\ q_N & p_N \le \varphi \le 1 \end{cases}$$

An illustrative example (1)

- A survey of water demand patterns in **596** domestic washrooms referred
- average number of users per washroom = **4.2** (head count, or denoted as 'hd').

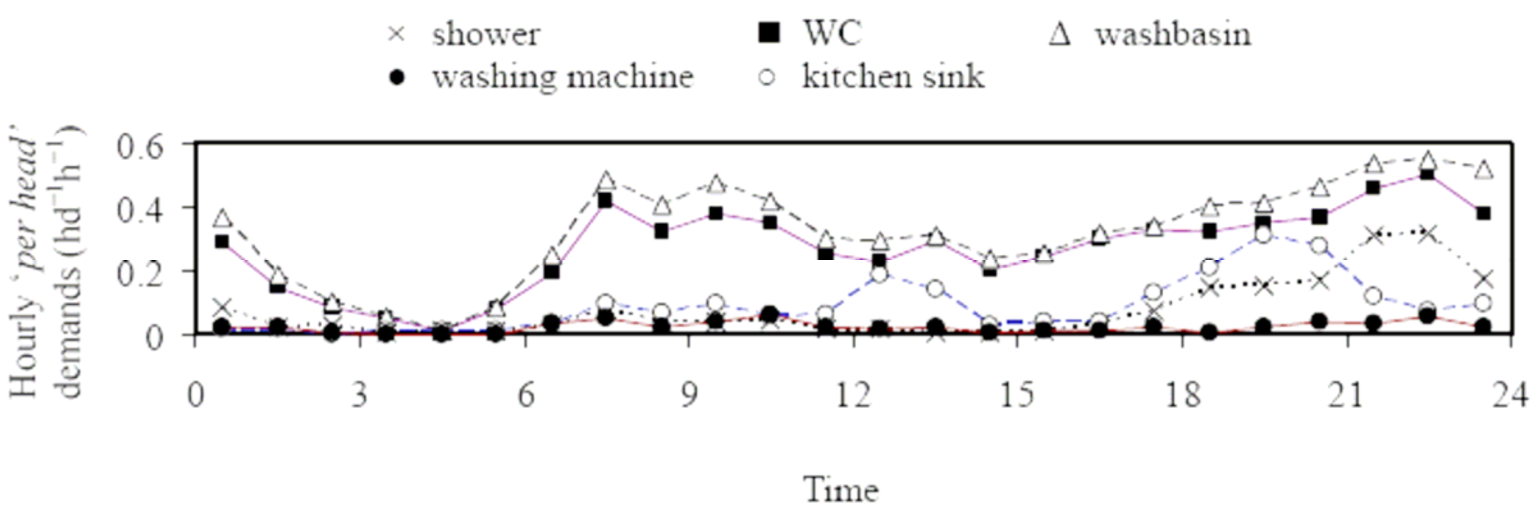


Figure 1: Diurnal demand patterns of some domestic appliances

An illustrative example (2): Typical demand patterns of domestic washroom appliances

Survey parameter	Normal distribution		Geometric distribution	
	AM (ASD)	Goodness-of-fit p-value	GM (GSD)	Goodness-of-fit p-value
Water closet (WC)				
Cistern volume V_{wc} (L)	9.2 (1.1)	>0.01	9.3 (1.1)	<0.0001
Operating time τ_{wc} (s)	101 (55)	<0.0001	85 (1.9)	≤0.0001
Washbasin				
Flow rate q_{wb} (L/s)	0.17 (0.07)	<0.0001	0.15 (1.6)	≥0.02
Operating time τ_{wb} (s)	16.0 (18.6)	<0.0001	10.7 (2.4)	≥0.002
Shower				
Flow rate q_{sh} (L/s)	0.31 (0.07)	<0.02	0.31 (1.2)	≥0.89
Operating time τ_{sh} (min)	13.7 (7.5)	≥0.0001	11.3 (2.0)	<0.0001

An illustrative example (3): Input parameters

- demand for all appliances between midnight and 5:00 a.m. was very low;
- peak '*per capita*' hourly demands occurred in the evening:
- WC:
  - 6:00 p.m. to 9:00 p.m. - average 0.59/hd/h (ASD=0.20/hd/h)
    - maximum 1.25/hd/h;
- Washbasin:
  - 8:30 p.m. - average 0.59/hd/h (ASD=0.21/hd/h)
    - maximum 1.25/hd/h;
- Shower:
  - 7:00 p.m. - average 0.18/hd/h (ASD=0.09/hd/h)
    - maximum 0.29/hd/h.

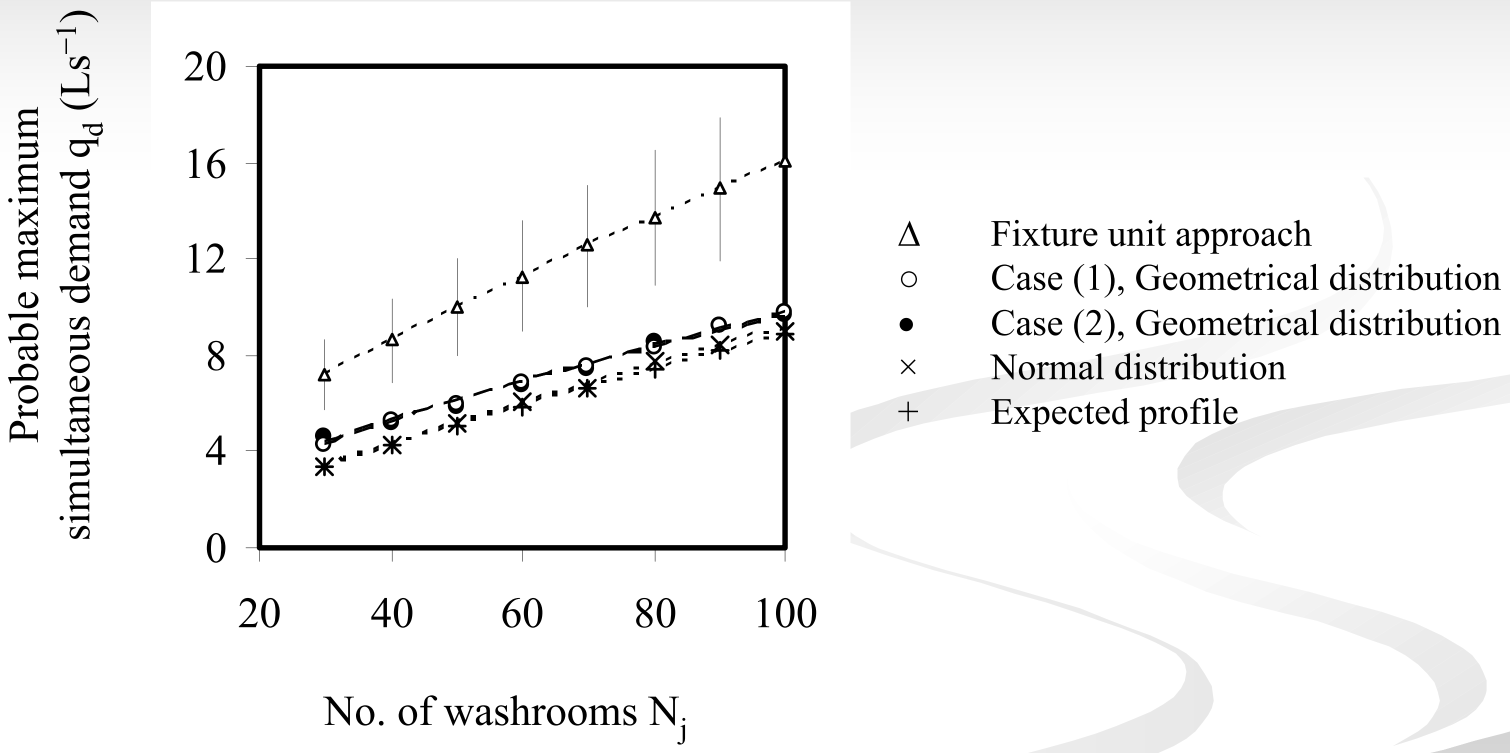
An illustrative example (4): Input parameters

- For simulations, e.g. a washroom:
- For appliances i=3 (i.e. a WC, a washbasin and a shower)
- assuming their respective peak demand profiles with an average occupant load per appliance of 4.2 hd (ASD=1.0 hd)

An illustrative example (5): Input parameters

- Based on the time-flow rate approach, the design flow rates q_d (L/s) due to a number of domestic washrooms were evaluated via Monte Carlo simulations for two cases:
  - **(1) none of the three appliances in a washroom would be used at the same time;**
  - **(2) WC cistern and washbasin in a washroom would be used at the same time.**
- Employing the arbitrarily selected profiles from 2000 simulated washrooms, together with the expected appliance profiles
  - expected flow rate $q_i = \langle \tilde{q}_i \rangle$
  - expected operating time $\tau_i = \langle \tilde{\tau}_i \rangle$
  - expected occupant load $N_p = \langle \tilde{N}_p \rangle$
  - during the recorded peak period $n_i = \tilde{n}_i(t)\big|_{\max(N_{o,i})}$

Simulation results (1)



Concluding remarks (1)

- Presently, hypothetical simultaneous usage patterns of domestic washroom appliances are used in the design of a water supply system which might not optimize the estimated demand for water supply system in some high-rise residential buildings of Hong Kong.

Concluding remarks (2)

- This study proposed a stochastic model of water demands in domestic washrooms for some high-rise residential buildings in Hong Kong using a time-flow rate approach while taking account of the simultaneous usage patterns of occupant loads, per-occupant demand rate, water flow rate and demand time of installed appliances

Concluding remarks (3)

- With an illustrative example for Hong Kong case, this paper presented a template for the development of a stochastic demand model that estimates the probable maximum simultaneous water demands for high-rise residential buildings

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