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Recent Issues and Developments in Burn-in Models

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Contents

1. Introduction

- Burn-in procedure is a manufacturing technique that is intended to eliminate early failures of the system or product.

- To burn-in a component or system means to subject it to a period of simulated use prior to the time when it is to actually be used.

-Those components (systems) which fail during the burn-in procedure will be scrapped and only those which survive the burn-in are put into field operation.

- An excellent survey of various research in burn-in can be found in Block and Savits (1997).

A survey of burn-in research with emphasis on mixture models, criteria for optimal burn-in and whether it is better to burn-in at the system or component level has been given.

- After Block and Savits (1997), there has been much development on burn-in models : (i) burn-in procedures for repairable systems have been developed (ii) a generalized assumption on the failure rate of the system has been proposed, and (iii) a stochastic model for burn-in procedure in accelerated environment has been developed.

- In this paper, recent advances and developments in burn-in models are surveyed in detail and some issues to be studied in the future study will be briefly discussed.

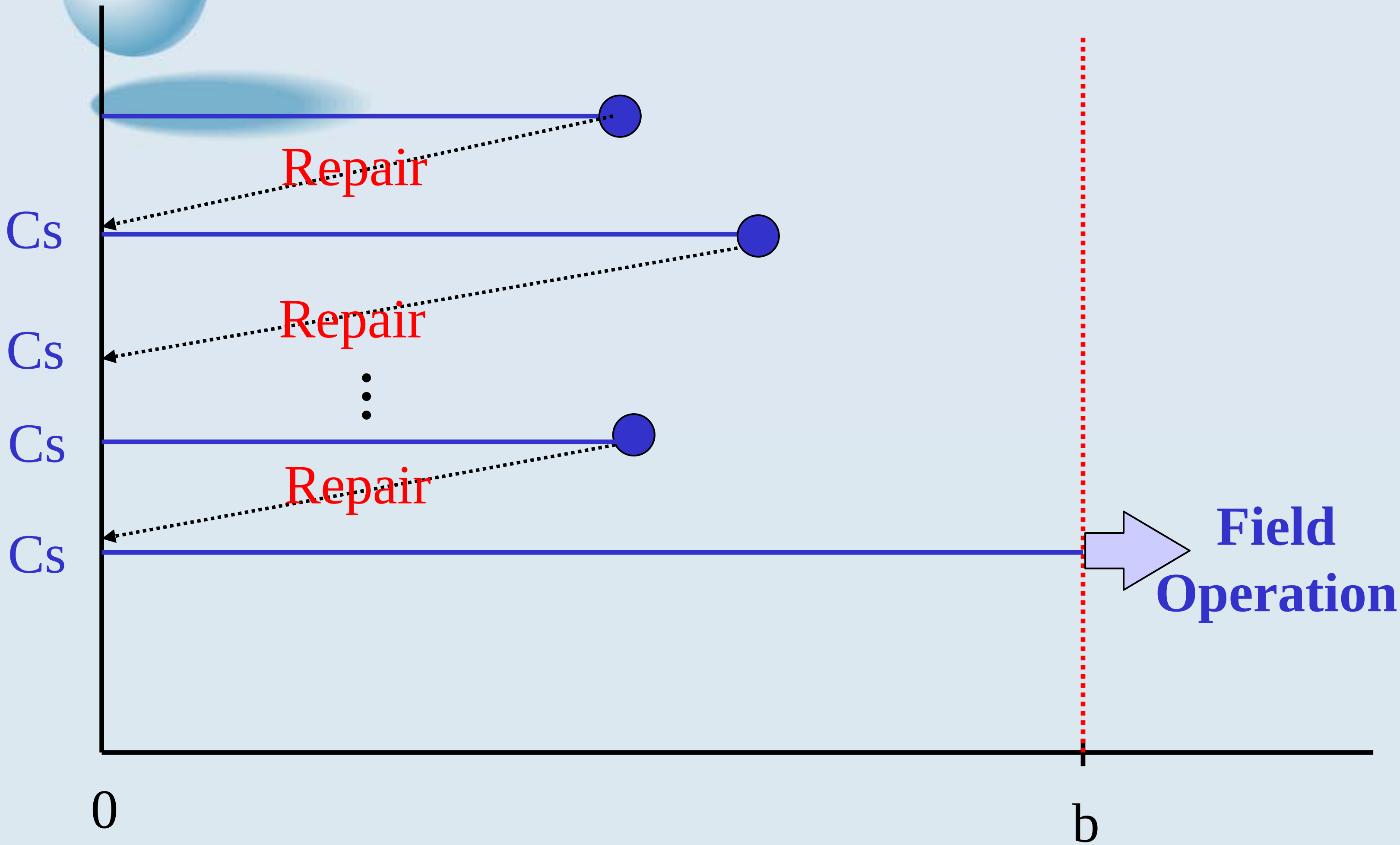
2. Recent Development of Burn-in Models

2.1 Burn-in Procedures for Repairable Systems

- Recently, burn-in procedures for repairable systems are proposed and studied.

● Burn-in Procedure A (Mi, 1994) : Consider a fixed burn-in time b and begin to burn-in a new component. If the item fails before a fixed burn-in time b, replace it with shop replacement cost Cs, and continue the burn-in procedure for the replaced item. If the item survives the burned-in item then the item is to be put into field operation. The cost for burn-in is assumed to be proportional to the total burn-in time with proportionality constant c_0.

Burn-in Procedure A



: Failure of Component, Complete Repair

Cs : Shop Repair Cost

- In field operation, Mi (1994) considered two kinds of replacement policies for non-repairable and repairable components, respectively : for non-repairable component, age replacement policy is considered and, for repairable component, block replacement policy is considered.

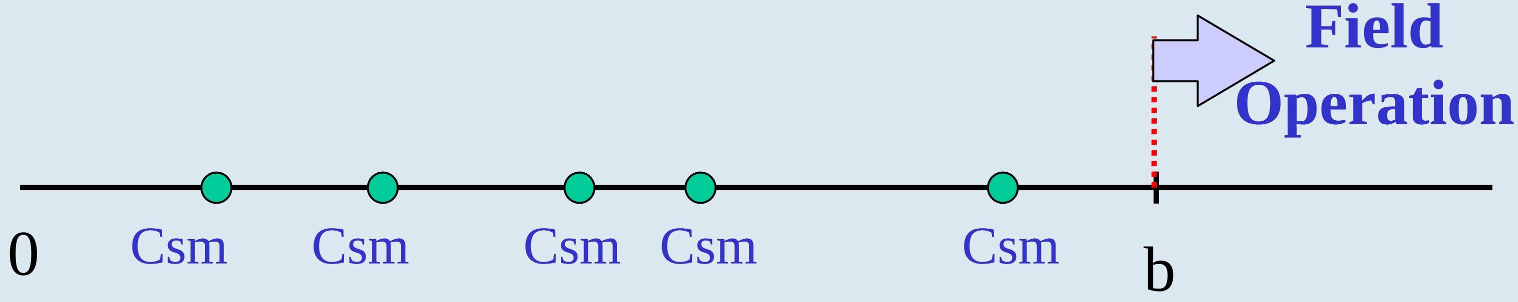
-By age replacement policy, a component is always replaced at the time of failure or T hours after its installation, where T is a fixed number, whichever occurs first.

-By block replacement policy, the component is replaced at time kT(k=1,2,…), where T is also a fixed number. For intervening failures, only minimal repairs are performed.

Considering these burn-in and replacement models, Mi (1994) obtained the long-run average cost rate $c_1(b,T)$ and studied the properties of the optimal (b^*,T^*) which minimizes $c_1(b,T)$.

● Burn-in Procedure B (Cha, 2000) : Consider a fixed burn-in time b and begin to burn-in a new component. On each component failure, only minimal repair is done with shop minimal repair cost Csm>0 (with Csm<Cs), and continue the burn-in procedure for the repaired component. Immediately after the fixed burn-in time b, the component is put into field operation.

Burn-in Procedure B



● : Failure of Component, Minimal Repair

Csm: Shop Repair Cost

Adopting the above Burn-in Procedure B, block replacement policy with minimal repair at failure is adopted as it was in Mi (1999). First, Cha (2000) obtained the long-run average cost rate $c_2(b,T)$ and showed that

$$c_2(b,T) \leq c_1(b,T),\ \forall 0 < b < \infty,\ 0 < T < \infty$$

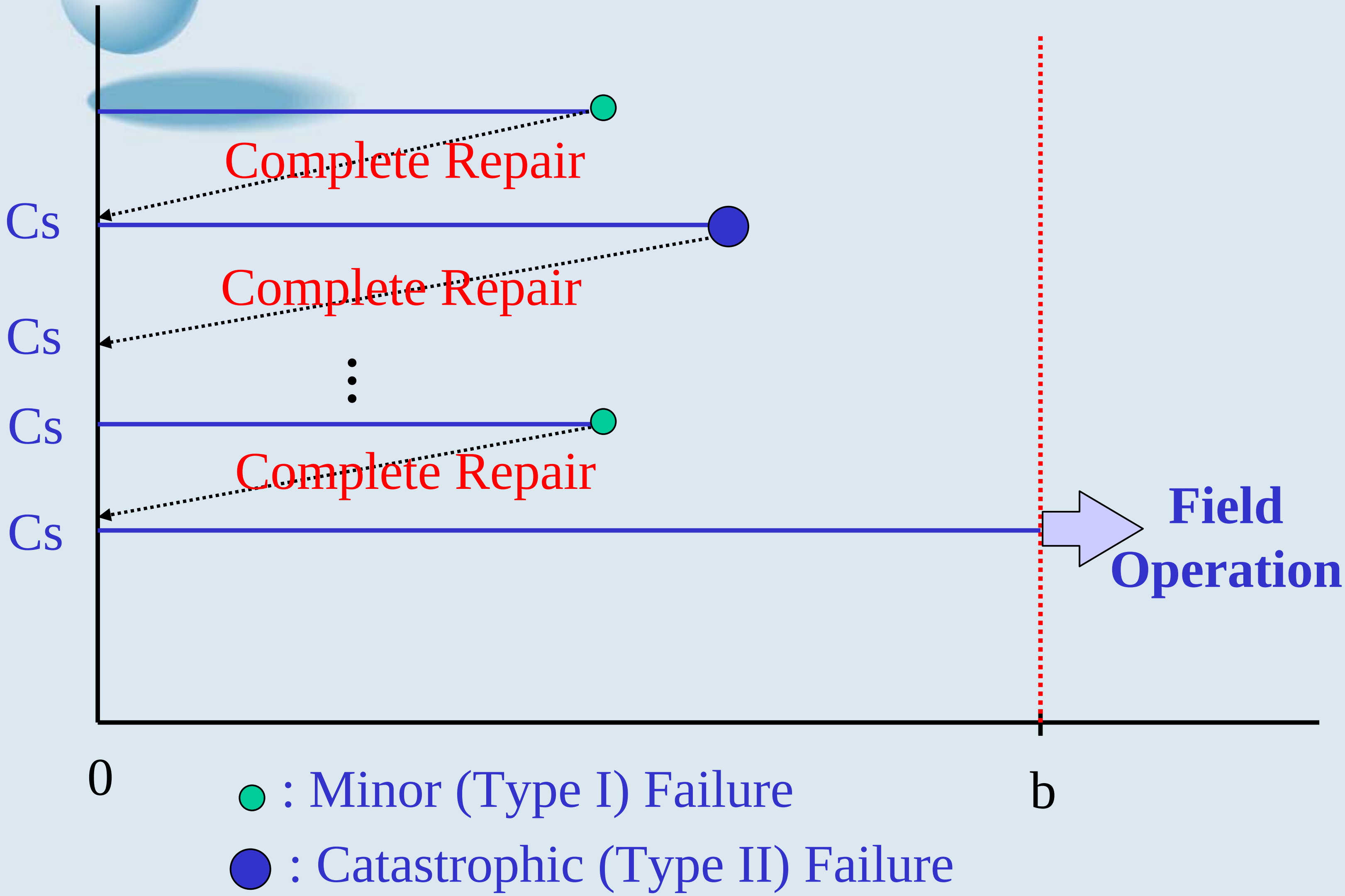
This inequality implies that Burn-in Procedure B is always preferable to Burn-in Procedure A when minimal repair method is applicable during burn-in process.

- The properties of optimal (b^*,T^*) which minimizes $c_2(b,T)$ are investigated.

● General Failure Model : when the unit fails, Type I failure occurs with probability 1-p and Type II failure occurs with probability p, $0 \leq p \leq 1$. It is assumed that Type I failure is a minor one thus can be removed by a minimal repair or a complete repair (or a replacement), whereas Type II failure is a catastrophic one thus can be removed only by a complete repair.

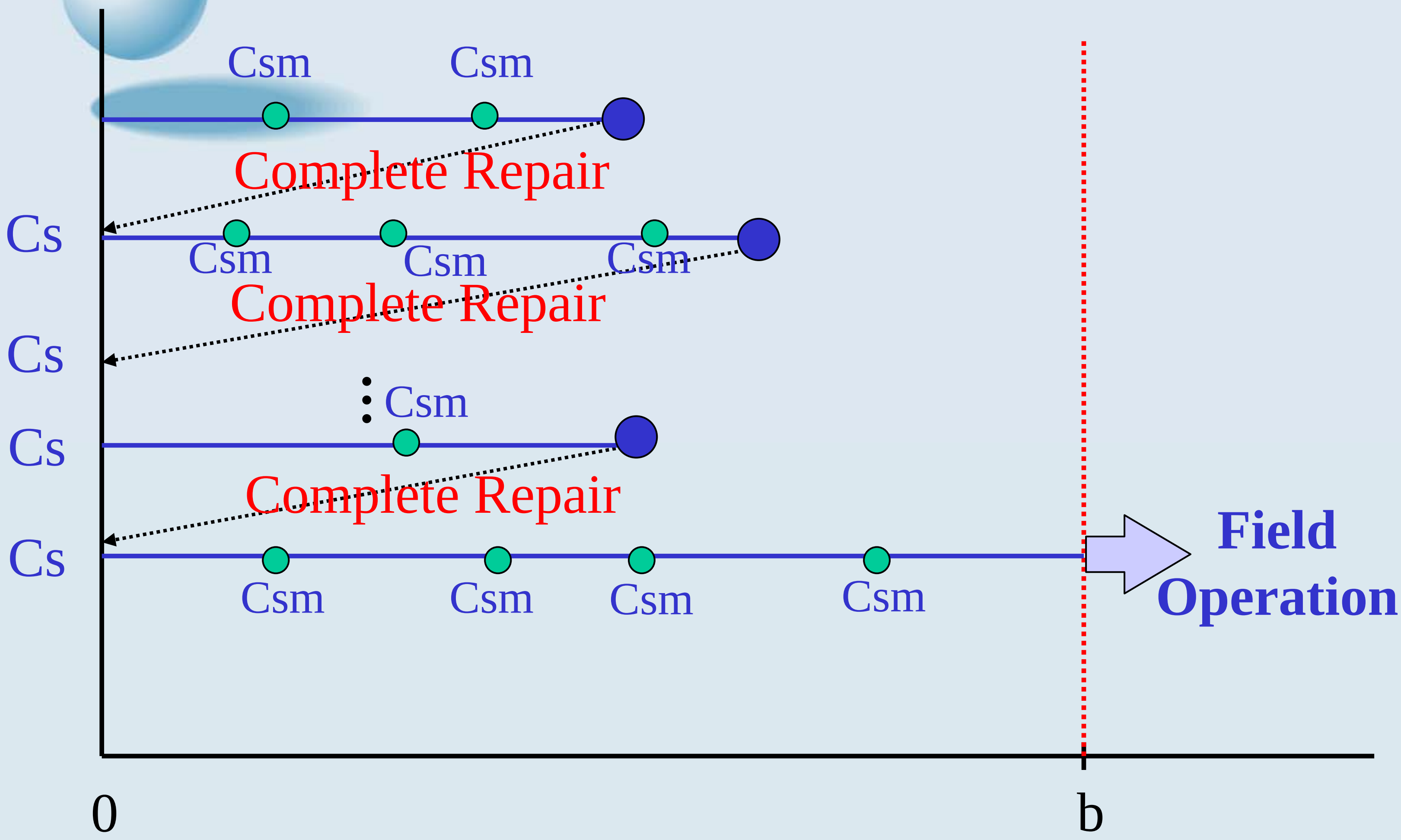
● Burn-in Procedure C (Cha, 2001) : Consider a fixed burn-in time b and begin to burn-in a new component. If the item fails before burn-in time b, then repair it completely regardless of the type of failure with shop complete repair cost Cs, and then burn-in the repaired component again, and so on.

Burn-in Procedure C



- Burn-in Procedure D (Cha, 2001) : Consider a fixed burn-in time b and begin to burn-in a new component. On each component failure, only minimal repair is done for the Type I failure with shop minimal repair cost (with Csm<Cs), whereas a complete repair is performed for the Type II failure with shop complete repair cost Cs. And continue the burn-in procedure for the repaired component.

Burn-in Procedure D



● : Minor (Type I) Failure, Minimal Repair

● : Catastrophic (Type II) Failure

-For both Burn-in Procedures C and D, in the field use, the component is replaced by a new burned-in component at the 'field use age' T or at the time of the first Type II failure, whichever occurs first. For each Type I failure occurring during field use, only minimal repair is done.

- Note that these burn-in and replacement models are generalizations of those of Mi (1994) and Cha (2000).

⇨ For these models, it is shown that

$$c_4(b,T;p) \le c_3(b,T;p), \ \forall 0 < b < \infty, \ 0 < p < 1$$

⇨ This inequality implies that Burn-in Procedure D is always preferable to Burn-in Procedure C when minimal repair method is applicable during burn-in process.

- The properties of optimal (b^*, T^*) are investigated.

-In Cha (2003) the generalized burn-in and replacement model considered by Cha (2001) is further extended to the case in which the probability of Type II failure is time-dependent. In the extended model here, when the unit fails at its age t, Type I failure occurs with probability 1-p(t) and Type II failure occurs with probability p(t), $0 \leq p(t) \leq 1$.

2.2 Generalized Assumption on the Failure Rate Function

● Bathtub Shaped Failure Rate Assumption

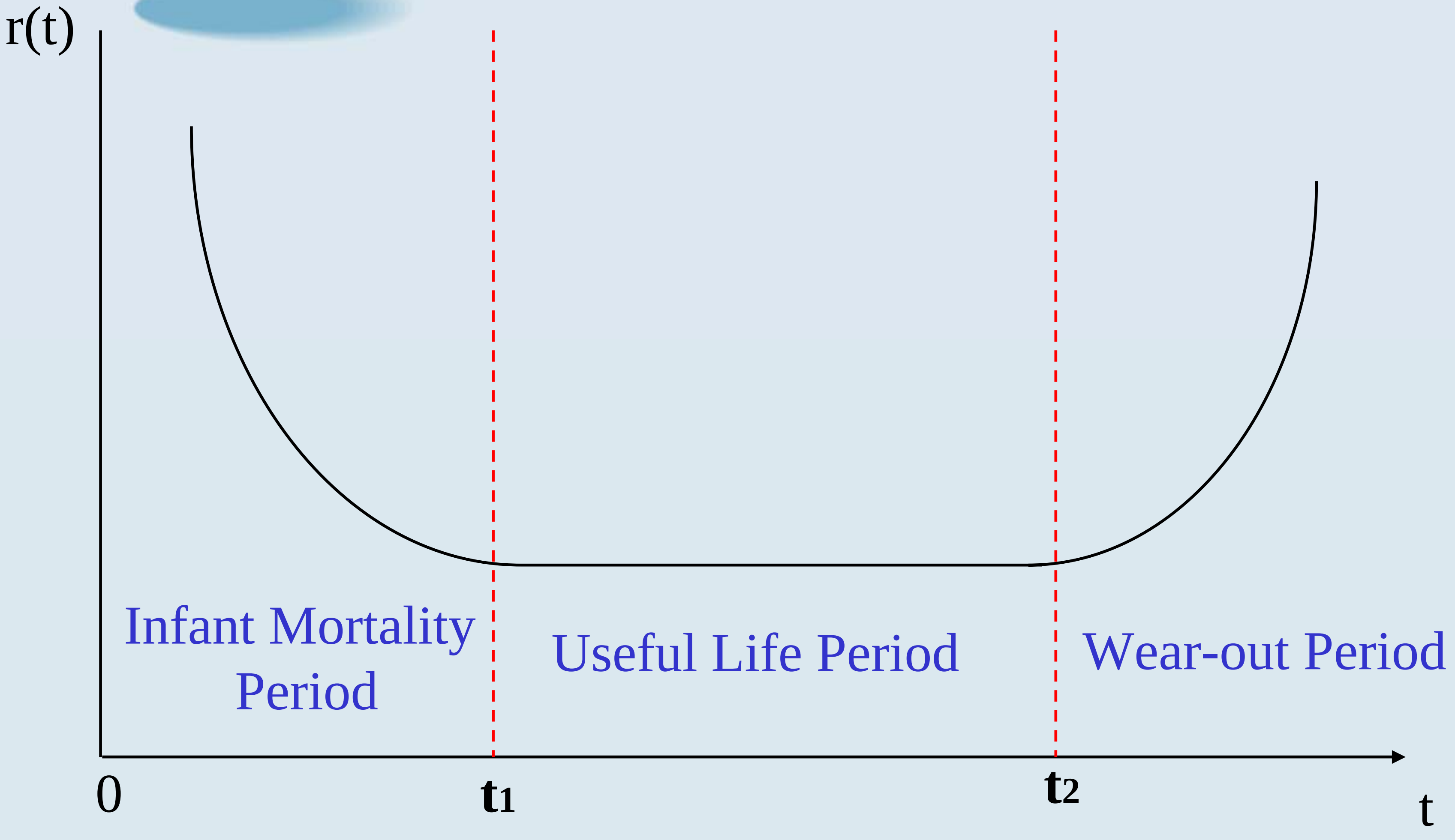
- It has been widely believed that many products, particularly electronic products or devices, exhibit bathtub shaped failure rate functions

Definition 1(Bathtub Shaped Failure Rate Function)

A failure rate function(FR) $r(t)$ is said to have **bathtub shape** if there exists $0 \leq t_1 \leq t_2 \leq \infty$ such that

$$r(t) = \begin{cases} \textit{strictly decreases}, & \text{if } 0 \leq t < t_1, \\ \textit{is a constant, say } \lambda_0, & \text{if } t_1 \leq t < t_2, \\ \textit{strictly increases}, & \text{if } t_2 \leq t. \end{cases}$$

- Much research on burn-in have been done under the assumption of bathtub shaped failure rate function.

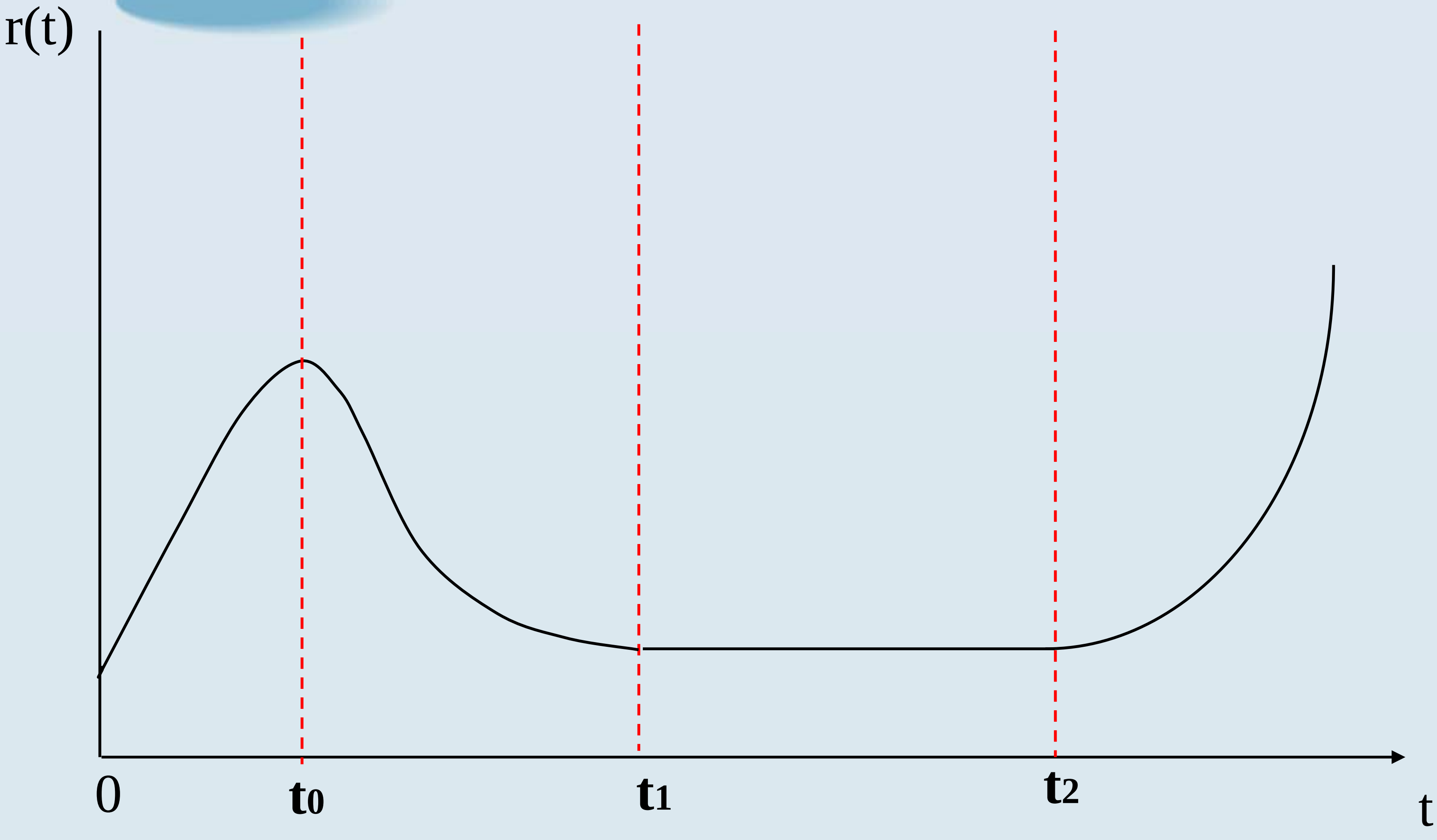

r(t)
Infant Mortality Period
Useful Life Period
Wear-out Period
0
t1
t2
t

- In addition to the traditional bathtub shaped failure rate function, there is also the so-called Modified Bathtub Shaped Failure Rate Function.

Definition 2(Modified Bathtub Shaped Failure Rate Function)

A failure rate function $r(t)$ is said to have **modified bathtub shape** if there exists $0 \le t_0 \le t_1 \le t_2 \le \infty$ such that $r(t)$ is strictly increasing in $t \in [0, t_0]$, and has a bathtub shape with change points t_1 and t_2 on the interval $[t_0, \infty)$.

- The modified bathtub shaped failure rate can be obtained from mixture of a distribution of strong component and that of weak component. (cf. Jensen and Petersen (1982))


r(t)
0
t_0
t_1
t_2
t

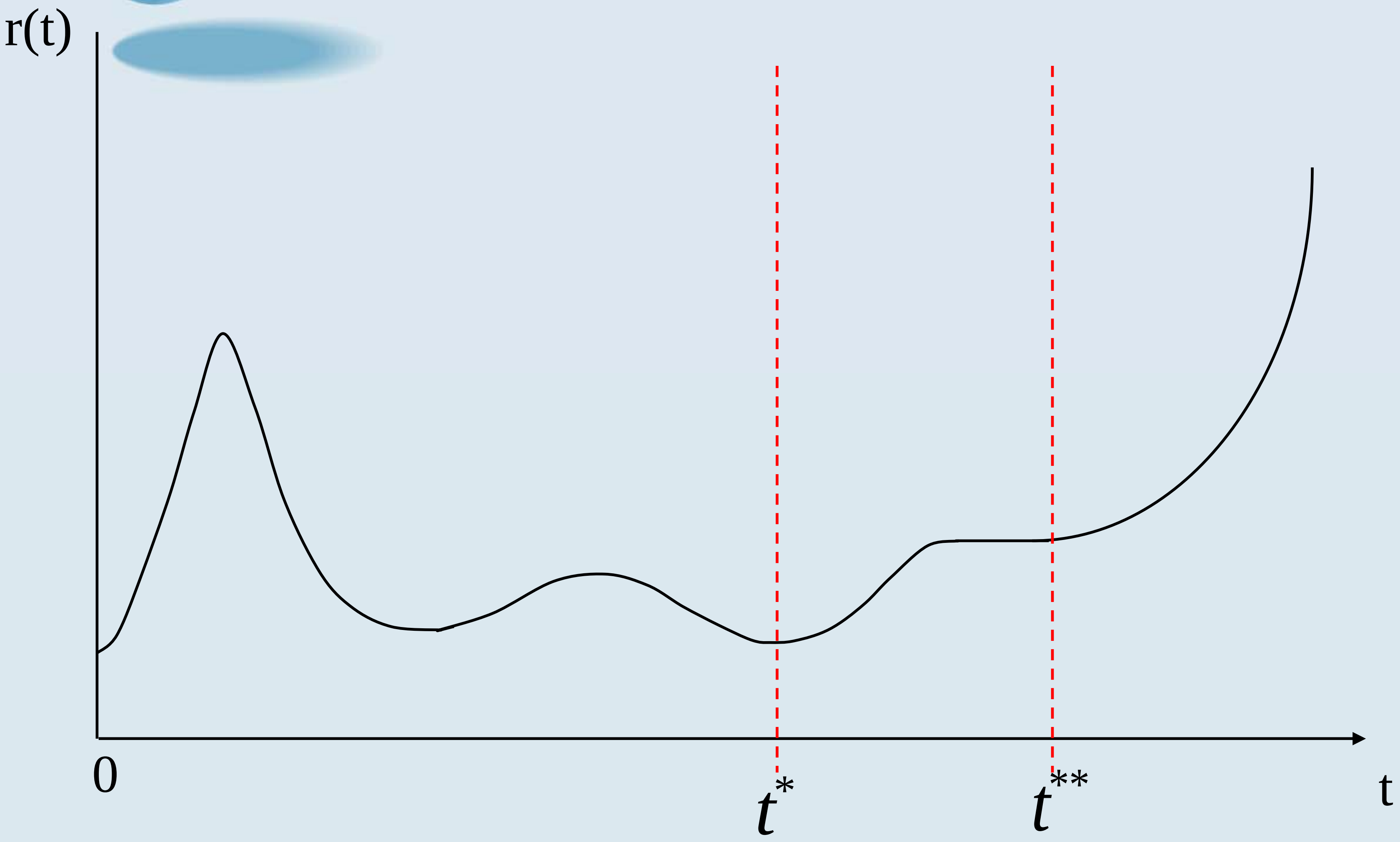
Generalized Assumption

- Although the assumption of bathtub shaped failure rate function is adopted in the most of researches on burn-in, it has been pointed out that it could be a rather restrictive assumption (see, for example, Klutke et. al. (2003))

-Especially, Kececioglu and Sun (1995) asserts that the bathtub shaped failure rate function describes only 10% to 15% of applications.

Definition 3 A failure rate $r(x)$ is eventually increasing if there exists $0 \le x_0 < \infty$ such that $r(x)$ strictly increases in $x > x_0$. For an eventually increasing failure rate $r(x)$ the first and second wear-out points t^* and t^{**} are defined by

$$\begin{aligned} t^* &= \inf\{t \ge 0 : r(x) \text{ is nondecreasing in } x \ge t\} \\ t^{**} &= \inf\{t \ge 0 : r(x) \text{ strictly increases in } x \ge t\}. \end{aligned}$$


r(t)
0
t^*
t^{**}
t

Note that

(i) if r(x) has bathtub shape with change points $t_1 \le t_2 < \infty$

(ii) if r(x) is modified bathtub shaped failure rate with

$$0 \le t_0 \le t_1 \le t_2 < \infty,$$

then it is eventually increasing with $t^* = t_1$ and $t^{**} = t_2$.

Therefore, the eventually increasing FR includes both the traditional bathtub shaped and the modified bathtub-shaped failure rate as special cases.

Recently, Cha (2006a, 2006b) considered optimal burn-in under the assumption of eventually increasing failure rate function and it has been shown that t^* in eventually increasing failure rate function plays the same role as t_1 in the bathtub shaped failure rate function.

2.3 Burn-in Procedures in Accelerated Environment

-Burn-in is generally considered to be expensive and the length of burn-in is typically limited. Furthermore, for today's highly reliable products, many latent failures or weak components require a long time to detect or identify. Thus, as remarked in Section 8 of Block and Savits (1994), burn-in is most often accomplished in an accelerated environment in order to shorten the burn-in process.

-However, nevertheless, much research has been done only on the burn-in procedures performed in the usual level of environment and there have been few probabilistic or stochastic approaches to the burn-in procedures in accelerated environment.

-Recently, Cha (2006b) proposed a new failure rate model for accelerated burn-in procedure and considered optimal accelerated burn-in time. The probabilistic frame for accelerated burn-in procedure in Cha (2006b) employs the basic statistical property commonly used in accelerated life tests (ALT).

- Notations

X : the lifetime of a component used under the usual level of environment

$F(t)$, $f(t)$, $r(t) = f(t)/\bar{F}(t)$: Cdf, pdf, FR of X

X_A : the lifetime of a component operated in the accelerated level of environment

$F_A(t)$, $r_A(t)$: Cdf, FR of X_A

(1) Lifetime During Accelerated Burn-in

During accelerated burn-in, we assume the followoing AFT Regression Model

$$F_{\mathcal{A}}(t) = F(\rho(t)), \ \forall t \geq 0$$

where

$\rho(t) \geq t, \ \forall t \geq 0$, $\rho(0) = 0$ and $\rho(t)$ is strictly increasing, differentiable function.

⇒ Failure Rate during Accelerated Burn-in

$$r_{\mathcal{A}}(t) = \frac{\rho'(t) f(\rho(t))}{1 - F(\rho(t))} = \rho'(t) r(\rho(t)).$$

(2) Lifetime After Accelerated Burn-in (Field Operation)

Right after a new component has been burned-in during a fixed burn-in time b in accelerated environment, the 'virtual age', which is transformed to the usual level of environment, of the component would be not less than b.

X_b : the lifetime of the burned-in component with accelerated burn-in time b

It is assumed that

$$P(X_b > t) = \exp\left(-\int_0^t r(a(b)+u)du\right) = \frac{\bar{F}(a(b)+t)}{\bar{F}(a(b))} = P(X > a(b)+t \mid X > a(b)), \ \forall\, t \geq 0.$$

Here, $a(b) \geq b, \ \forall b \geq 0$, $a(0) = 0$ and $a(b)$ is strictly increasing, differentiable function.

⇨ The performance of a component burned-in under accelerated environment during (0,b] is the same as it were operated in the usual environment during (0,a(b)] .

Failure Rate in Field Operation

The burned-in component with accelerated burn-in time b and ‘field use age’ u has failure rate

$$r(a(b)+u),\ \forall\, u \geq 0$$

Combining the accelerated burn-in phase and the field use phase, the failure rate function of component with accelerated burn-in time b, which is denoted by $\lambda_b(t)$, can be expressed as

$$\lambda_b(t) = \begin{cases} \rho'(t)r(\rho(t)), & \text{if } 0 \leq t \leq b \text{ (Burn-in Phase)}, \\ r(a(b)+(t-b)), & \text{if } t > b \text{ (Field Use Phase)}. \end{cases}$$

Recently, Cha (2006b) and Cha and Na (2007) investigated optimal accelerated burn-in time based on the above model.

3. Some Other Topics for Developments

● Burn-in for components whose degradation can be described by a Wiener Process

In Tseng et al. (2003), it is assumed that the product failure corresponds to the first passage time of the degradation path beyond a critical value and a Wiener process is used to describe the continuous degradation path of the quality characteristic of the product. Assuming different degradation patterns for weak and normal populations, the problem of determining the optimal burn-in time has been considered.

Adopting shock as a burn-in operation

In Finkelstein and Esaulova (2006), the non-asymptotic and asymptotic properties of mixture failure rates in heterogeneous populations are studied. It is pointed out that, in a specific setting, a shock performs a kind of burn-in operation. Thus burn-in problem can be considered in the model.

- Optimal burn-in in multi-dimensional optimization problems

Since the failure rate function of a system used in field operation depends on the burn-in procedure it experiences, it is thus natural to take burn-in and other operating characteristics (e.g. maintenance policy) into consideration all together at the same time. Recently, there has been some research which considers multi-dimensional optimization problem with burn-in time as one of design variables. For example, Cha and Mi (2008) considered three-dimensional optimization problem, where optimal burn-in time, optimal work size and optimal replacement policy are determined for a data processing system.

4. Concluding Remarks

-In most cases, produced product generally have high initial failure rate and thus burn-in is very important research area of reliability.

-In this paper, recent advances and developments in burn-in models have been surveyed.

-Furthermore, some issues and research topics which can be developed through future study have been suggested.

-The issues and ideas suggested in this paper could also be applied to the problem of determining optimal software testing procedure.