Reliability assessment basing on importance measures

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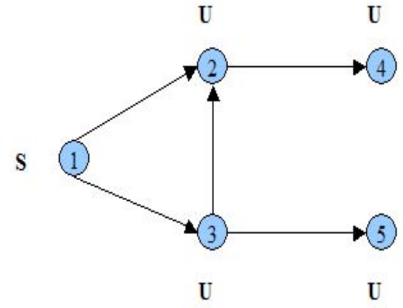
INTRODUCTION

Purposes

Our aim is to propose a general approach to assess the structure of a networked system, considering that:

- its performances must be evaluated with respect to each user node (Local performances) and to the whole network (Global performance);
- Importance measures must be used carefully because a networked system is a multi-statuses one and a large set of them are not additive;
- there are relationships among different Importance measures so that they could provide the same ranking of elements or different ones;
- “traditional” approaches, which are based on the explicit identification of the system structure functions or on the enumeration of its statuses, become obsolete as the network becomes large.

Networked system



Random graphs made up of unfaultable (Nn) user and (Ns) source nodes, connected by (Nc) ”directed”, “binary” edges.

Status of each edge: $x_j = 0$ if the edge is available, 1 elsewhere

Status of each node: $x_i = 0$ if the (user) node is connected to at least a source trough at least a path made up of available edges, 1 elsewhere

A weight w_i is assigned to each user node according to the disutility produced when it is not connected to a source at least .

The <u>Global performance</u> of a networked system can be evaluated by the sum over its nodes of the <u>Local performance</u> (probability that the node is not connected to at least a source), multiplied by the relevant weight:

$$U = \sum_{i=1}^{Nn} w_i U_i .$$

See “Risk significance importance measures for a networked system”

General information

The unavailability of a system (risk metric) can be written as

$$U = U_j \cdot U^{j+} + (1 - U_j) \cdot U^{j-} = (U^{j+} - U^{j-}) \cdot U_j + U^{j-}$$

where $U^{j+} = \Pr\left[\Phi\left(\ldots, x_j = 1, \ldots\right) = 1\right]$ and $U^{j-} = \Pr\left[\Phi\left(\ldots, x_j = 0, \ldots\right) = 1\right]$

Different Importance measures are proposed for probabilistic risk assessment:

Birnbaum measure $$\sigma_j = \frac{\partial U}{\partial U_j} = U^{j+} - U^{j-}$$

Criticality importance measure $$I_{cr} = \sigma_j \cdot \frac{U_j}{U}$$

Risk Achievement Worth $$RAW_j = \frac{U^{j+}}{U}$$

Risk Reduction Worth $$RRW_j = \frac{U}{U^{j-}}$$

Differential Importance measure $$DIM_j = \frac{\dfrac{\partial U}{\partial U_j}}{\sum_i \dfrac{\partial U}{\partial U_i}}$$

All the measures are local ones: they deal with the modification of one parameter at a time, considering the related maximum increase and decrease in risk , while the other parameters are fixed at their nominal values.

For a networked system, the Differential Importance measure must be referred to its Global performance.

DIM for the edge j with respect to the whole network results:

$$*DIM_j = \frac{\sum_{i=1}^{Nn} w_i \cdot \frac{\partial U_i}{\partial U_j}}{\sum_{j=1}^{Nc} \sum_{i=1}^{Nn} w_i \cdot \frac{\partial U_i}{\partial U_j}}$$

DIM for the edge j with respect to the user node l results:

$$*DIM_j^l = \frac{w_i \cdot \frac{\partial U_i}{\partial U_j}}{\sum_{j=1}^{Nc} \sum_{i=1}^{Nn} w_i \cdot \frac{\partial U_i}{\partial U_j}}$$

See "Risk significance importance measures for a networked system"

The Risk impact curve represents a linear relationship between U^{j*}/U and U_j^*/U_j :

$$\left(\frac{U^{j*}}{U}\right) = \alpha \cdot \left(\frac{U_j^*}{U_j}\right) + \frac{1}{RRW_i}$$

where

U_j^* is the new value of the edge j unavailability

U_j is the current value of the edge j unavailability

U^{j*} is the system unavailability when the edge j unavailability is at its new value.

U is the current value of the system unavailability

$$\alpha = I_{cr} = \sigma_j \cdot \frac{U_j}{U} = \left(* DIM_j . \sum_i \sigma_i \right) \cdot \frac{U_j}{U} = U_j \cdot \left(RAW_j - \frac{1}{RRW_j} \right)$$

Methodological approach

Networked system analysis is performed by means of:

- *MonteCarlo simulation,*
- *Cellular Automata.*

MonteCarlo techniques are adopted in order to simulate the evolution of the system configuration by means of an “indirect approach”: sampling the time at which the transition occurs and the last failed / repaired element.

Cellular Automata are adopted in order to “solve” the graph (without identifying the system structure functions or enumerating its statuses) by means of the verification of the existence of (at least) a path between (at least) a source node and each user node.

Birnbaum measures are computed for each edge, with respect each user node, basing on U_i^{j+} *and* U_i^{j-}.

They provide all the significant information:

- Local and Global performance of the network;
- ranking of the edges with respect to each user node and to the whole network;
- relationships among different importance measures (Risk impact curve).

In order to investigate the network structure, …

… we assume the same weight for all user nodes $w_i = 1$ *;*

In this case, the Global performance of the network is the sum of its Local performances.

… we assume have the same unavailability for all edges $U_j = 0{,}5$ *.*

This means to assume:

- the same value for the failure and repair rates (assuming an exponential distribution for the failure and repair time);
- the same probability for each status of the system.

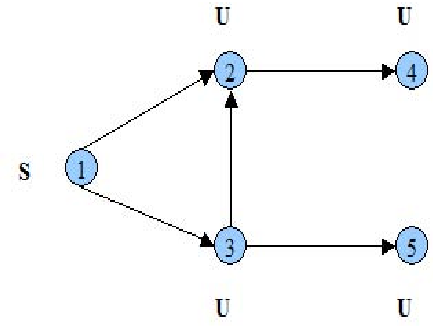
Under this assumption, variance reduction techniques are not necessary.

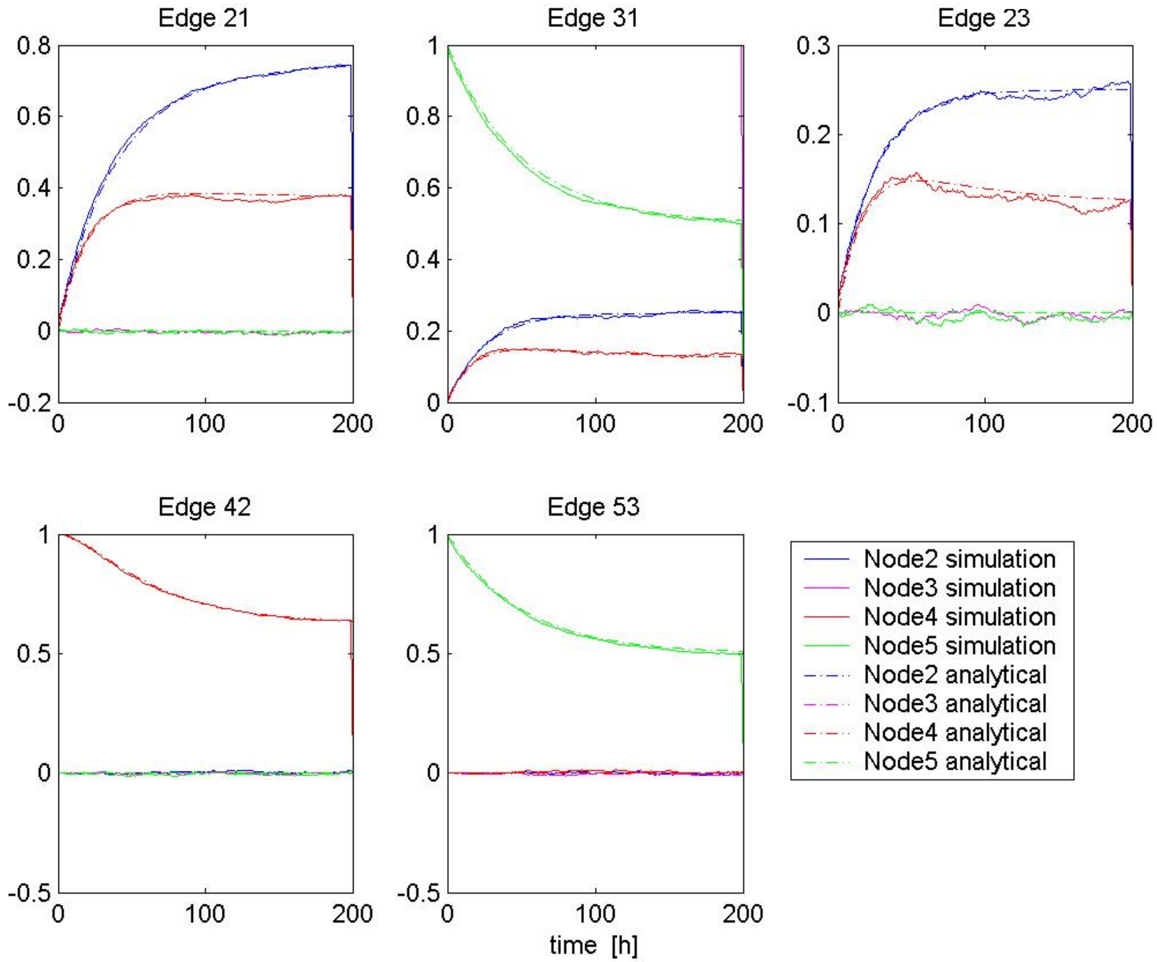
They are necessary under real assumptions for the failure and repair rates of edges, in order to improve the sampling of rare events, generally due:

- to the order of magnitude of the transition rates with respect to the mission time,
- to the different order of magnitude of the failure and repair rates,
- to the high redundancy degree which generally characterizes a networked system.

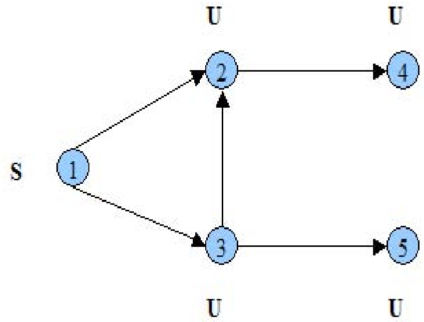
APPLICATION CASE

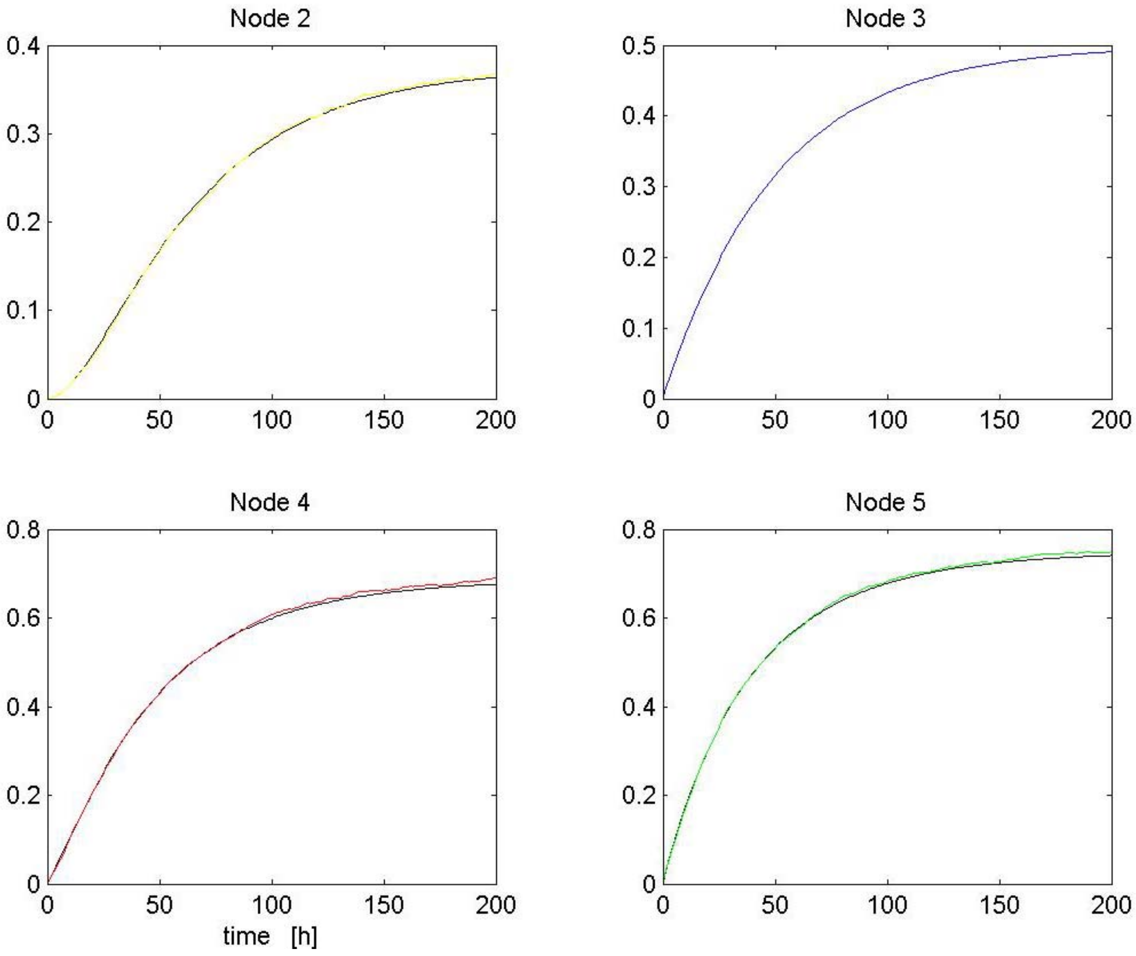
Birnbaum measures





Local performances

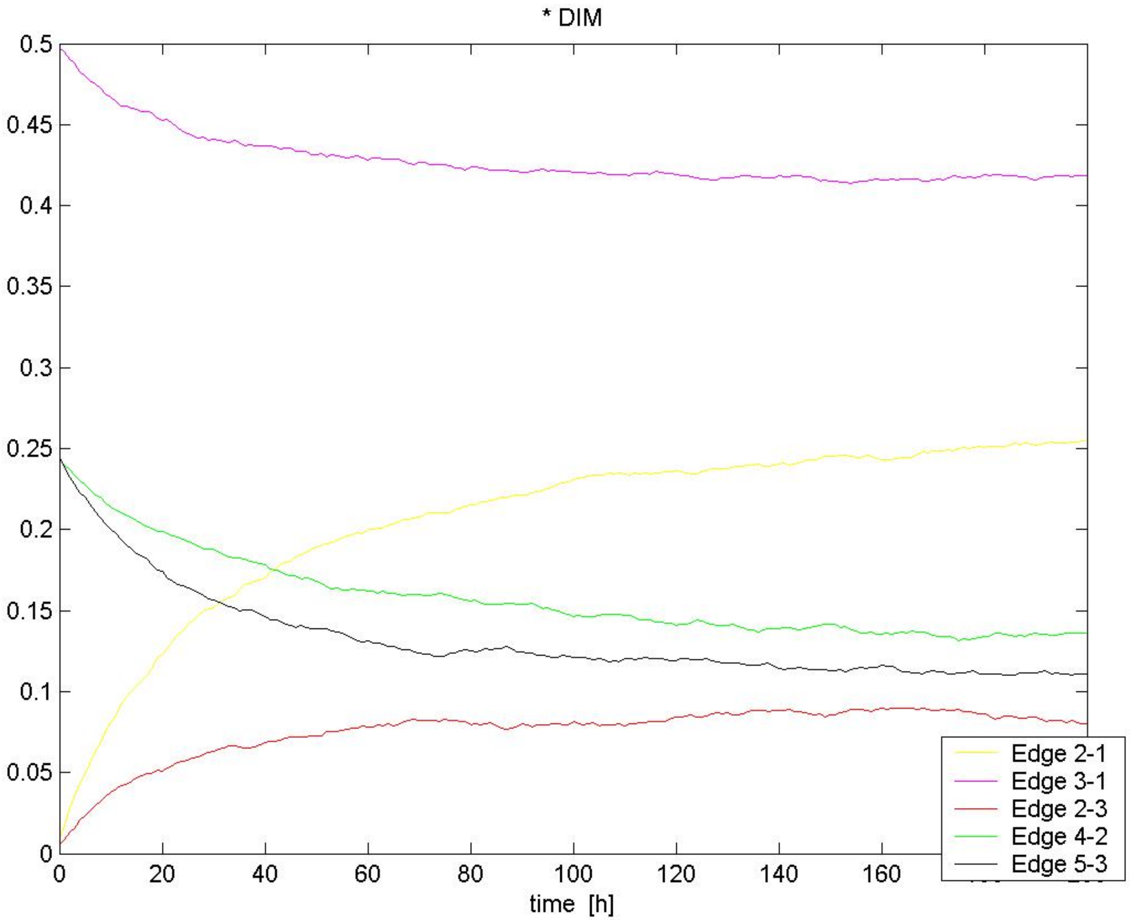


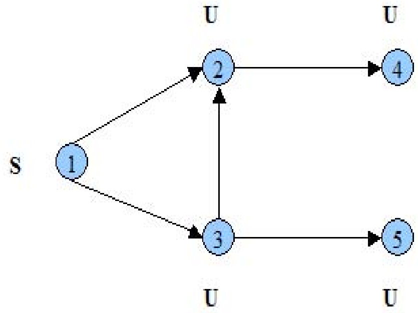


Node	Local performance*
2	0,3750
3	0,5000
4	0,6875
5	0,7500

* Stationary values analytically computed

Differential Importance measures

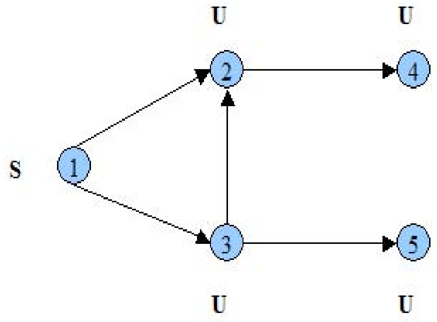


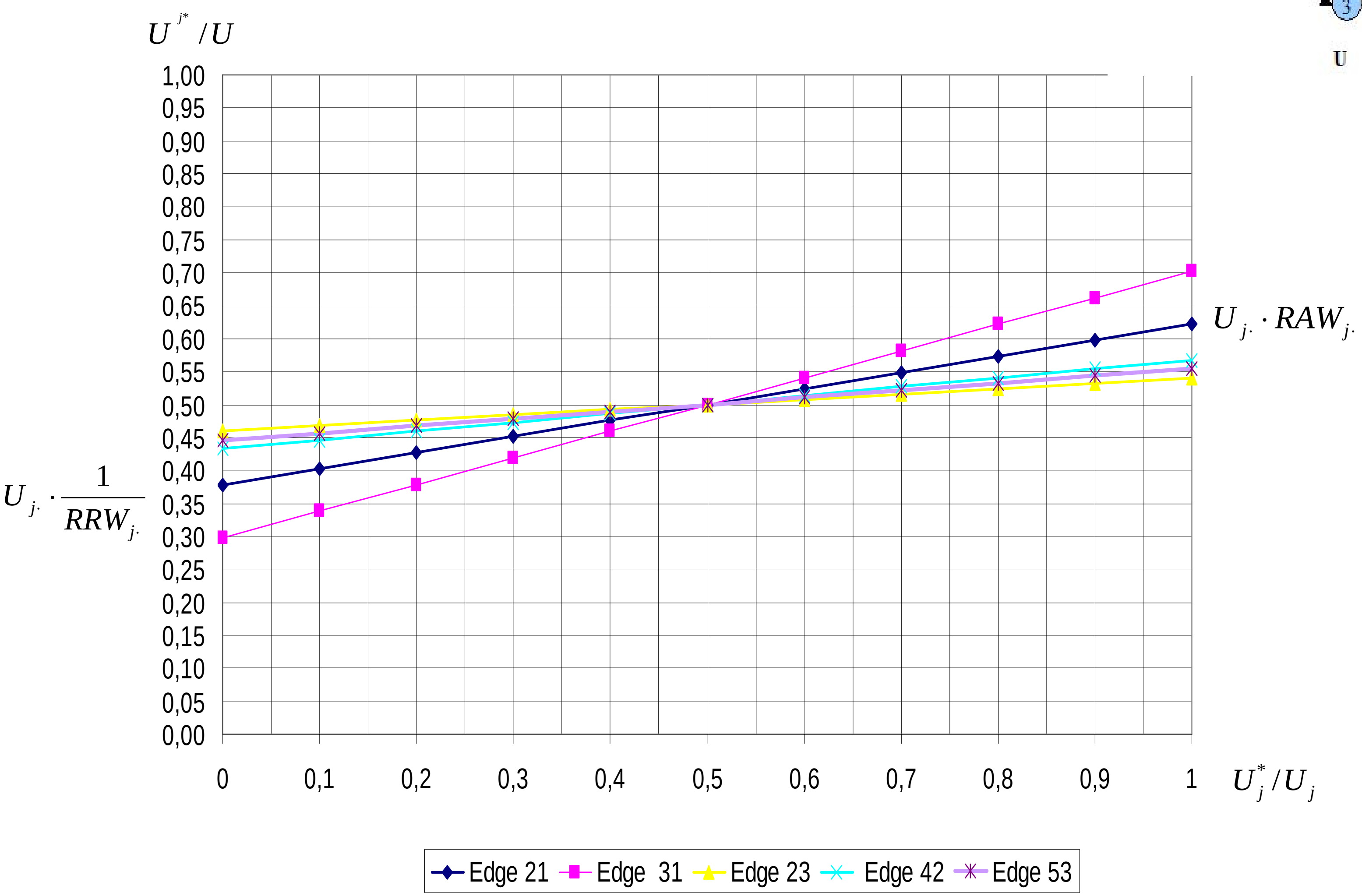


Rank	Edge	DIM*
1	31	0,42
2	21	0,25
3	42	0,14
4	53	0,11
5	23	0,08

* Stationary values analytically computed

Risk impact curve





CONCLUSION

The reliability / availability of a networked system must be assessed with respect to each user node (Local performances) and to the whole network (Global performance).

A networked system can be “solved” by Monte Carlo simulation and Cellular Automata, without the identification of the system structure functions or the enumeration of its statuses.

The structure of a networked system can be assessed by assuming the same unavailability for all edges and the same weight for all user nodes.

The elements of the system must be ranked by means of an importance measure which is referred to the Global performance of the network.

All the importance measures can be referred to the Global performance of the system;
they provide the same ranking of elements with respect to the structure of the network .

The same approach can be used assuming different values for the unavailability of the edges and/or for the weight assigned to user nodes.

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